The Multi Intruder "Brazil Nut" Problem

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Abstract

The "Brazil Nut" Problem is the effect of larger particles in a bed of smaller granular particles rising to the surface when the container is shaken to some extent. Up till now most of the experiments on the Brazil Nut Problem used only a single intruder. This report deals also with the multi intruder Brazil Nut Problem in order to investigate the effects of putting in more than one intruder. These effects are the possible interactions occurring between the intruders, either attracting or repelling. Therefore causing the intruders to go down, rise to the surface or stay at the depth they were put in. These interactions can also vary for every type of intruder used, causing different behavior for different kind of intruders.

Most of the industries involved in granular materials have to deal with more than one larger granular particle in a bed of smaller particles. So, these industries are interested to know and understand the interactions going on in such a system under different circumstances and initial conditions.

The most important varied parameters of the system were the density and the size of the intruders. For all experiments applies that the effects of increasing the density or size were more pronounced in 0.5mm glass beads as background material than in 1mm glass beads.

The density dependence result for 0.5mm glass beads showed that the rise time had a peak around $\rho/\rho_m \approx 0.5$, which was three times higher than $T_{asymptote}$. For 1mm glass medium the rise time had only a small unstable region at $\rho/\rho_m \approx 0.5$, but the rise time can be considered as slightly increasing.

In the size dependence experiment (for both glass beads) we observed for single as well as multi intruders far from the density peak: a larger single intruder or larger configuration, acting as a compound, rises faster. If the single or multi intruder configuration was situated on the density peak (nylon) the same rise time for all sizes was found in 1mm glass beads. In the 0.5mm case the larger single intruder rises slower and the multi nylon experiment was highly unstable (not considered as a compound anymore).

From the multi intruder experiment using different sizes of individual intruders the next 'rule of thumb' to obtain the same rise time with different size steel intruders was found:

1 1" intruder ~ 1.5 ¾" intruders ~ 3.1 ½" intruders

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Preface

A 3 months internship is compulsory in my studies Applied Physics at the University of Twente and one can do it either in the Netherlands or abroad. I started looking for an internship available in the United States, Great Britain or Germany in November 2001. So I wrote application letters to several companies, institutes and universities, but the majority didn't respond at all and the rest declined…

So mid January 2002 I switched my interest to some Dutch companies to see if an internship was available in the Netherlands. Unexpectedly I suddenly heard via Harmke van Aken that our professor of the Department of Fluids, Detlef Lohse, had an internship available in Chicago.

Contact was made very quickly with Heinrich Jaeger and in the end everything worked out, so I started working on the Brazil Nut Problem on June 24th. Shaking balls and beads for the summer…

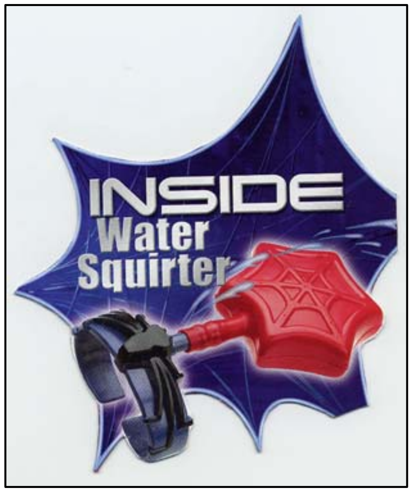

Fig 1: Water squirter in cereal

During my stay in Chicago I was almost occupied with granular materials 24/7; in the morning I ate my bowl of cereals while applying the "Brazil Nut" effect to my box of cereals. My new knowledge of this effect made it possible to get out the Spider-man water squirter and play first with it even before eating!!!

In my leisure time with the many hot weekends I was occupied with granular materials too: working against it while playing beach volleyball at one of the beaches at the North Side of Chicago! Or just lie down in the granular material called 'sand' getting a tan.

The large variety of experiences with granular materials I had was very entertaining wherever I was. The atmosphere at the Department of Physics in the Jaeger/Nagelgroup was very encouraging, so I really enjoyed doing research on "the Multi Intruder 'Brazil Nut' Problem" in my lab.

Introduction to Granular Matter

A few examples of granular matter are sand, salt, sugar or any grain like material in general. These granular materials contain unique behavior with respect to fluids for example. This is very clear when you consider a binary fluids system. When you stir such a system it causes the two fluids to be homogeneous afterwards (if the fluids contain good dissolvability).

However, when you shake a system containing two different sizes of granular materials, they will separate in a way in which the larger particles have moved to the surface. So instead of being homogeneous size segregation took place in the container with granular materials!

This size segregation seems to be contradicting the rule of increasing entropy, but with granular material the system is far from equilibrium and dynamical effects compensate any thermodynamical considerations. So that is the reason why this segregation in granular media is not a contradiction to this entropy rule [1].

Besides this different behavior in the fluid state of granular material, this medium can also act as a solid or a gas. You can see this fact on the beach for example, because while walking from the beach to the sea the sand obviously behaves as a solid. This same sand can be poured from your left hand into your right, thus acting as a fluid. Sand can also act as a ‘gas’ for example when the wind is severe and throwing sand particles in your eyes during a nice game of beach volleyball…

In all these states of matter, granular materials are somehow behaving differently from the usual solids, fluids or gases and are therefore often called the unofficial “fourth state of matter”. The reason for this different behavior with respect to ordinary matter can be found in the fact that these granular systems can dissipate energy due to the frictional forces.

Many industries are involved in processing or using granular materials, so it is a major concern to reveal all the secrets of granular materials to efficiently work with these materials. It is clear for example that the pharmaceutical industry has to be careful while handling powders to produce pills, because every pill has to contain the same amount of effective substance. So size segregation is something the pharmaceutical industry wants to prevent. Other industries like mining, agriculture, food processing industry, construction industry and many others need to process grain like material too [1].

“Brazil Nut” Problem?!:

When I was younger I was always helping my mother with picking red currants and afterwards we had to pick the currants from the stalks. Every harvesting day we ended up with large buckets full of red currants and we shook the buckets to see if all the stalks were removed. I can still recall that all the ripe large red currants were on top of the pile, while all the little green or less ripe ones were on the bottom. Without knowing I was witness of size segregation caused by the Brazil Nut Problem!

But why is it called the *Brazil Nut* Problem then? Well, when you shake a can of mixed nuts for example, all the larger Brazil nuts can be seen on top of the pile of mixed nuts. This effect of larger particles rising to the surface in a shaken container with other granular material is therefore termed "Brazil Nut" Effect. This phenomenon is still not fully understood and that is why it is still the "Brazil Nut" *Problem* even though it has been studied since the 1930s [2]!

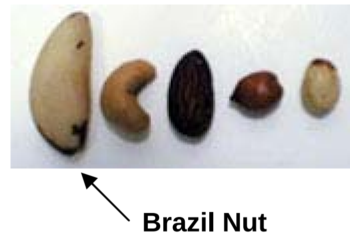

Fig 2: Mixed nuts including a Brazil Nut

The Brazil Nut Problem has been proposed to be attributed to several phenomena that could occur in a shaken container with granular material. These effects have to be regarded as still unconfirmed causes for the Brazil Nut Problem:

Percolation is the effect that the smaller granular particles can slip through the holes made by the larger ones and in this way causing size segregation [3].
Reorganization can be seen when a shake is applied to the system. The neighboring smaller particles are filling the gaps below the larger intruder, therefore causing size segregation [4,9].

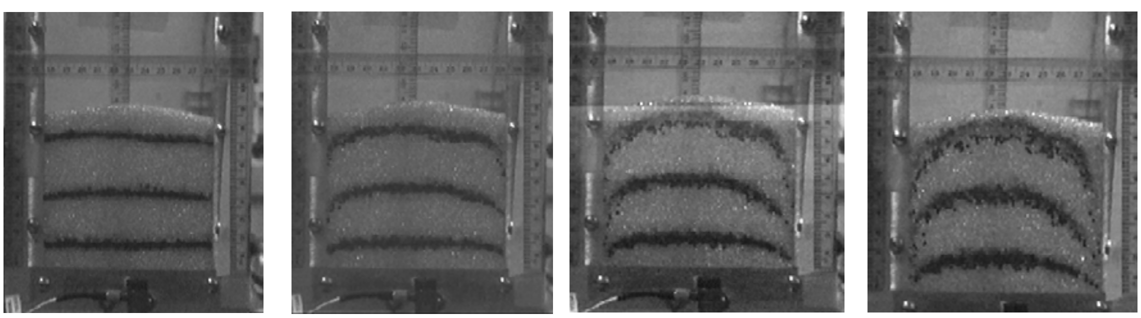

Fig 3: Convection in a semi-2D container [5] using 1mm glass beads with mustard seeds for visualization purpose only. This convection is likely the same in the 3D situation.

Convection can be seen in the shaken container going up in the center capturing the larger particles and going down in a very thin layer near the wall (figure 3). Therefore the larger particles are emerging in the center and are moving to the wall to try to go down with the flow again, but they cannot and are trapped on top of the surface due to convection [6,10]. The heap formed during shaking the granular matter is caused by this effect also (see figure 3).
Condensation could be occurring when you're shaking a binary granular system at a critical temperature, which depends on the ratio of mass and diameter [3]. Up till now this effect has only been observed in Molecular Dynamics (MD)

simulations on a computer. In these simulations the mass and diameter of the two granular materials could be controlled. By adjusting these parameters Hong et al. [3] were able to condensate either of the granular materials to the bottom. The phenomenon of larger particles segregating to the bottom is called "Reverse Brazil Nut Problem", but has only been observed in an experiment by Shinbrot and Muzzio [12]. Burtally et al. have observed something equal to this MD-effect in a mixture of bronze and glass spheres of similar diameters [11].

Unlike the fourth effect, the first three effects have been observed in particular experiments. But it is still uncertain whether one or a combination of these phenomena can be causing the larger particles to be rising to the surface.

Up till now only the single intruder Brazil Nut Problem has been investigated, in which one intruder was put in the middle of a pile of glass beads. In these experiments almost all of the important parameters have been changed: relative density, relative diameter, air pressure, roughness of the cylinder wall, size of the cylinder. My internship project was build around the multi intruder Brazil Nut Problem to investigate the effects of putting in more than one intruder. These effects are the possible interactions occurring between the intruders, either attracting or repelling. Therefore causing the intruders to go down, rise to the surface or stay at the depth they were put in. These interactions can also vary for every type of intruder used, causing different behavior for different kind of intruders.

Most of the industries mentioned earlier are working with more than one large particle in a smaller granular background too. So it is important to know the behavior of such a system before one can apply it to these industrial processes.

Experimental aspects

The experiments were performed in a 12cm inner diameter acrylic cylinder with 1mm glass beads glued to the walls. They were glued to the wall for two reasons; to impose a stable convection in the container induced by friction with the walls. Besides, the glued beads prevent the wall to get smoother while performing experiments. During the experiments the glass beads are scouring against the acrylic wall causing a changing roughness of the wall. So also for reproducibility reasons beads were glued to the wall. The cylinder was fixed to a VTS 100 electromagnetic shaker and placed on a heavy and stable table to prevent vibrational response of the table while shaking.

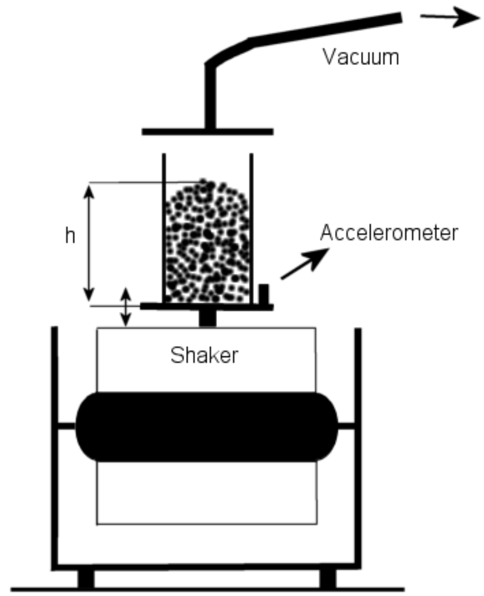

Fig 4: Setup used for all experiments

The setup used is shown in figure 4 and the cylinder was filled up to height *h* with one of the two sorts of glass beads used: *d=1mm* (ρ_m*=2.4 g/ml*) and *d=0.5mm* (ρ_m*=2.5 g/ml*) glass beads. Both glass beads were ranging ±10% in diameter and are considered to be mono-disperse. After filling the cylinder the whole system was leveled with the bolts of the VTS shaker to tune the convection rolls to be exactly axi-symmetric in the cylinder.

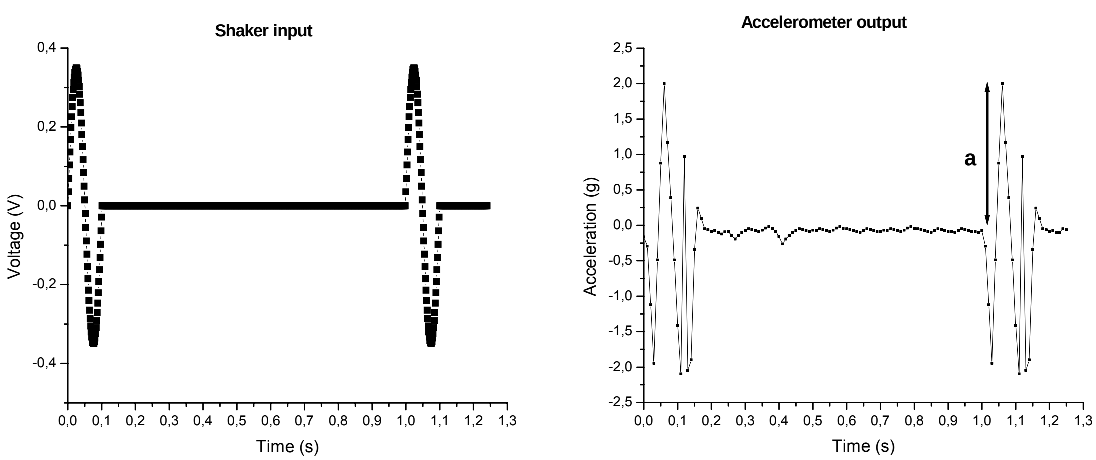

Fig 5: The input sinusoidal wave of the shaker (left) resulted in the output shown of the accelerometer (right). The piezo accelerometer calibration was 20.53 mV/g relative to the earth gravity field. The accelerometer output has been filtered out by a low pass filter (f_{cutoff}=175 Hz) in order to suppress the higher frequencies (noise) in the output signal. The peak after roughly 1 second in the accelerometer output is due to the glass beads falling back into the cylinder.

A PCB piezo accelerometer was fixed to the cylinders base to measure the acceleration of the system. The cylinder was shaken vertically once every second with a one period 10Hz sine wave called a 'tap'. This provides enough relaxing time for the system to settle before the next tap is applied. The shaker input and the output of the accelerometer can be seen in figure 5. Obviously the response of the cylinder to the sinusoidal input wave is not sinusoidal anymore due to the dynamic response of the electromagnetic shaker and the pile of beads falling back. The only thing that actually matters is that the response is constant and reproducible. This is the case with our system and therefore only the amplitude of the signal is important for the acceleration parameter:

$$\Gamma = \frac{a}{g} \qquad (1)$$

In this parameter is *a* the amplitude produced by the accelerometer (see figure 5) measured in magnitudes of $g=9.81 m/s^2$. In almost all of the experiments performed, the acceleration parameter was kept constant at $\Gamma=2.3$, for which the peak-to-peak value of the accelerometer output was used to determine Γ.

Before every experiment the cylinder is emptied to prevent the measurements to be affected by the ordering of the glass beads of the previous experiment. Then the cylinder is filled again and a certain number of spherical intruders (diameter, *D*, and density, ρ) are

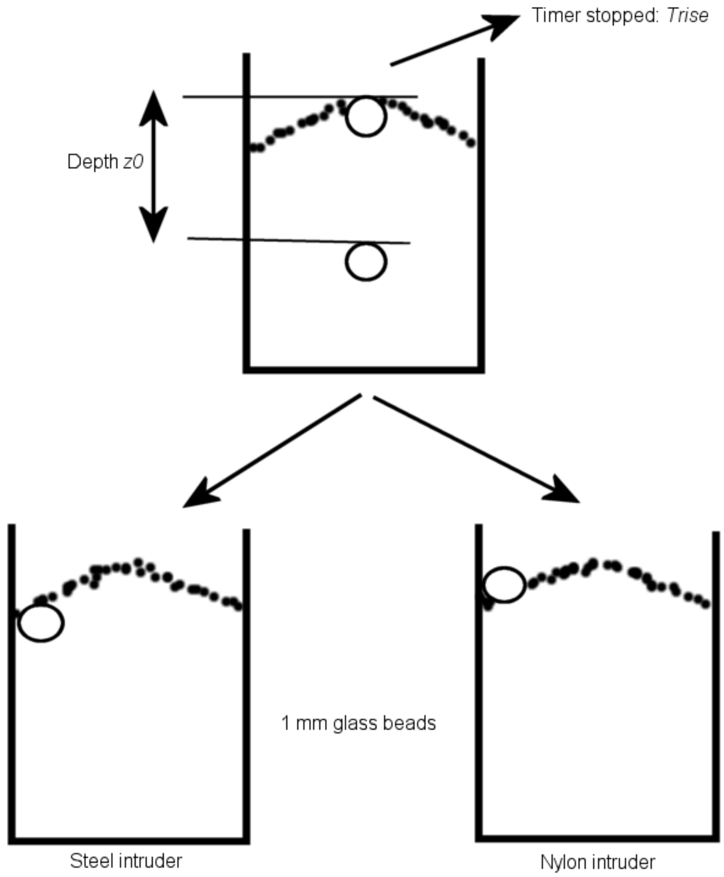

Fig 6: Definition of the rise time, T_{rise}, and the experimental problems using 1mm glass beads: the nylon intruder starts 'floating' to the wall right after emerging at the surface in the center. The steel intruder however, is emerging in the center or on its way to the wall or sometimes even after reaching the wall. Just before emerging a dimple can be seen in the cone surface. So for steel intruders this dimple is sometimes moving to the wall, after which the steel intruder emerges at the surface. This effect caused problems with the definition of the rise time in some experiments and will be mentioned later when these problems occurred.
In experiments in which we vacuumed the cylinder the problems in 1mm glass beads became even worse. For a pressure lower than roughly 20 kPa (20% of p_0) the steel intruder is not emerging anymore, while the nylon intruder had no problem surfacing.

placed at the designated depth z_0. After which the rest of the glass beads are poured on top of the configuration of intruders. Both the diameter and density of the intruders could be adjusted to investigate the Brazil Nut Problem with a vast variety of approaches.

The rise time, T_{rise}, of the intruders put at a depth z_0 is then determined with a stopwatch when the intruders emerge at the surface (see also figure 6). Because we were shaking with 1Hz the amount of time coincides with the number of taps applied to the system.

During some of the experiments the number of intruders had to be increased and this caused a small drop of the acceleration parameter, Γ. To account for this drop we increased the amplitude of the sinusoidal shaker input wave during these experiments to reassure that the shaking remained the same. After vacuuming the cylinder with beads to perform experiments at a lower pressure we also had to adjust the shaking, because of the missing weight of air. Therefore we had to decrease the amplitude of the input sinus-wave for the shaker for a large amount.

For both glass beads it was difficult to level the system, but it was most difficult for the 0.5mm glass beads. The system was very sensitive and the heap was always moving to the lowest point of the setup, in this way enabling us to adjust the system to get the heap centered again. Especially with 0.5mm glass beads the heap was wandering around the center during shaking. Every time we succeeded to level the system to perform our experiments.

Horizontal vibrations are also affecting the leveling of the shaker setup, but it did not affect our experiments: the horizontal vibrations were measured with the same PCB accelerometer and the horizontal acceleration can be considered as low. It can be a major concern when the shaking is well above $\Gamma=5$, but we were shaking with $\Gamma=2.3$ so this did not cause severe problems.

Results[1]

Several kinds of experiments have been performed with a single intruder as well as multiple intruders and they will be discussed in separate sections:

- Density dependence
- Size dependence
- Miscellaneous

Density dependence:

Regarding the density dependence several experiments were carried out in which first 1-inch (2.54 cm) intruders with different densities were used. In order to cover the whole density range 10 intruders ranging from foam to steel were used. The results are shown in figure 7 in which the relative density ρ/ρ_m is measured relative to the background medium.

The graphs shown in figures 7a and 7b demonstrate for 1mm glass beads that a small dependence on the relative density is present at atmospheric and lower pressure. The rise time (T_{rise}) is slightly increasing for both graphs with a little wiggle situated around $\rho/\rho_m \approx 0.5$. Around this wiggle the measurements were fluctuating more than elsewhere and is regarded as an unstable region.

The same range of different intruders was used in an experiment with 0.5mm background material, but besides this experiment we also used a hollow acrylic intruder. This intruder could be filled with lead to tune the density without affecting the roughness of the intruder. In this experiment 4 foam intruders, 12 measurements with an (hollow) acrylic intruder and 1 steel intruder were used to produce the graph shown in figure 7c. This result shows that when we were using 0.5mm glass beads, the little peak from the 1mm glass beads case is much more pronounced. Figure 7c is in agreement with the result of Möbius et al. [7] (depicted in figure 7d) when you consider the different boundary conditions. The overall higher rise time in figure 7c is due to a lower acceleration parameter and a larger diameter cylinder used in our experiments. When we take into account the results found in [7] and compare them with our results from figure 7c, we can assume that also in our system the density peak will vanish for a lower pressure (figure 7d).

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[1] For all of the results applies that every point shown in a graph is an average of at least 3 measurements, but with large fluctuating measurements at least 5 experiments were performed.

Furthermore, almost all of the error bars in the graphs are determined to be at least 5%, because of the fluctuating measurements and the systematic errors in the experiments. When the standard deviation of the experimental data is considered it is almost always below 5%, but when the data was very noisy the error bar percentage was increased.

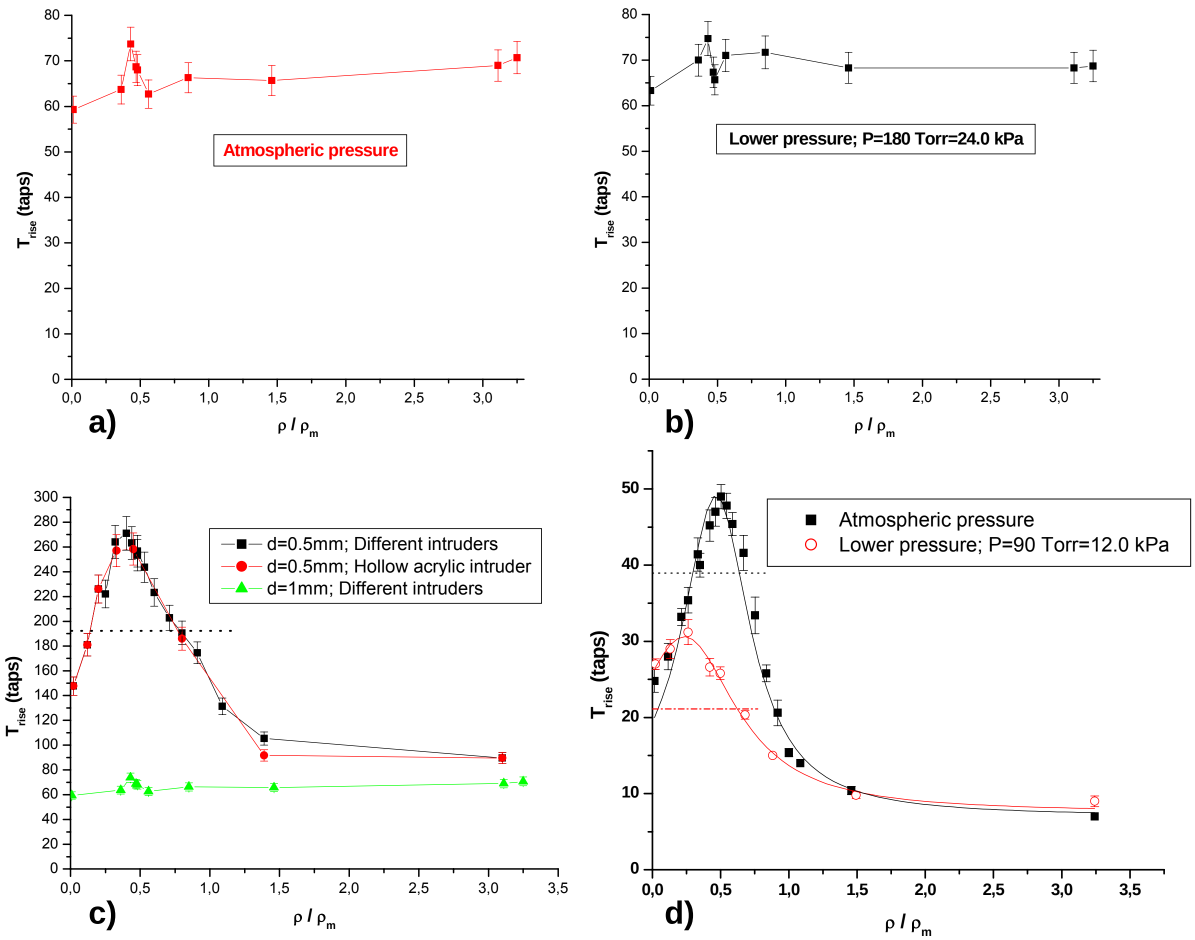


Fig 7: The density dependence is determined for intruders at a depth z_0=4.5 cm in the 12 cm diameter cylinder with filling height h=11 cm.

a) *1mm glass beads (D/d=25) at atmospheric pressure*
b) *1mm glass beads (D/d=25) at lower pressure; P=180 Torr=24.0 kPa*
c) *0.5 mm glass beads (D/d=51) at atmospheric pressure using different intruders to vary the relative density (■). In combination with a foam and steel intruder a hollow acrylic intruder was filled with lead in order to vary the density as well (●). The dotted line (black) shows the rise time of tracer particles in absence of an intruder. For comparison is the 1mm glass beads density result also included (▲).*
d) *0.5 mm glass beads (D/d=51) at atmospheric pressure (■) and lower pressure; P=90 Torr=12.0 kPa (○). These results of Möbius et al. [7] were determined using Γ=5, instead of Γ=2.3 in our experiments. A different cylinder (8.2cm diameter) and filling height (h=8.6) were used in their case. For both results the T_{rise} of the background particles is shown with the dotted and dot-dashed lines.*

From figure 7c it can also be observed that there is no difference in using many different intruders to vary the density or using the hollow acrylic intruder filled with lead. So the different roughness or restitution coefficient of the intruders does not affect the rise time, which confirms the result found in [7].

So for 0.5mm background material a much larger unstable region can be found than in the 1mm case. The rise time on the peak is roughly a factor 3 higher than $T_{asymptote}$, which is defined by the rise time of steel. Unlike the 3D situation, no peak in the density dependence can be observed in the semi-2D situation as can be seen in figure 8 [5]. These results of Niemuth et al. [5] are in agreement with Liffman et al. [8]. In 2D the rise time clearly decreases for denser intruders, where a slight increase can be observed in the 3D 1mm glass beads case.

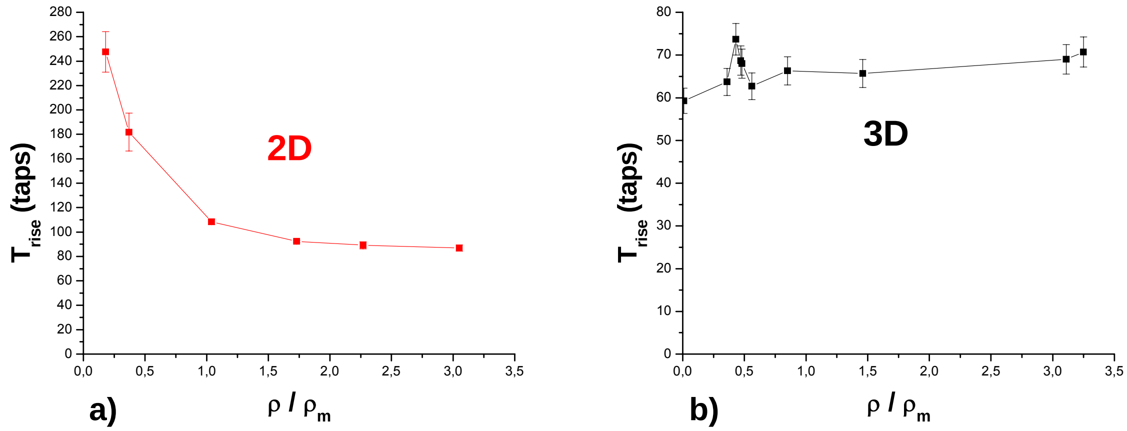


Fig 8: The density dependence in the semi-2D situation performed by Niemuth et al. [5] (figure 8a) in which the intruders ranged from thin rings to solid disks. In this way the density was varied in the semi-2D container with 1mm glass beads. The intruders were placed at a depth z_0=8.5 cm where the depth in the 3D case was set at z_0=4.5 cm (figure 8b).

<u>*Size dependence:*</u>

-Single intruder:

For both background materials (0.5 and 1mm glass beads) a single intruder size dependence experiment has been performed. This means that the diameter of the intruder put in the cylinder was increased. The results are shown in figure 9.

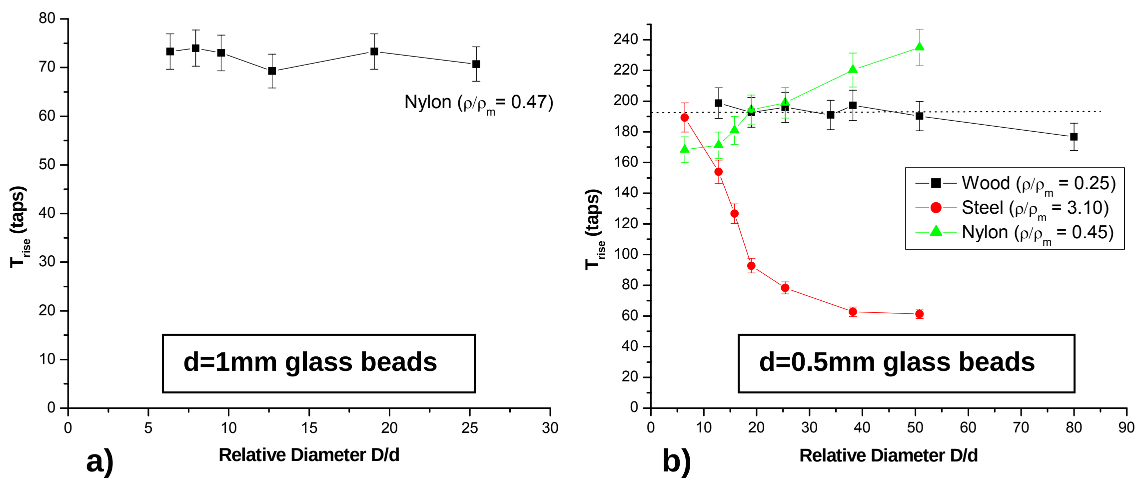


Fig 9: To determine the single size dependence the diameter of the intruder was increased for both background materials: in 1mm glass beads (figure 9a) only for nylon, but in 0.5mm glass beads (figure 9b) for wood (■), steel (●) and nylon (▲). Every intruder was placed at a depth z_0=4.5 cm and carried out under atmospheric pressure. The black dotted line in figure 9b is the T_{rise} of the background material.

As can be seen in figure 9a, the rise time of the different nylon intruders in 1mm glass beads remains the same. Where a significant increase is observed in the 0.5mm glass beads case. The result from figure 9b is in agreement with the size dependence in 0.5mm glass beads by Möbius et al. [7]. The different behavior of the same nylon intruders in a different medium could be found in the fact that nylon is right on the peak in the density dependence graph (figure 7). All occurring effects are always far more dramatic for 0.5mm glass beads than in 1mm background material.

When figure 9b is considered, it is striking that only the intruders situated on the density dependence peak ($\rho/\rho_m \approx 0.5$) are rising slower for an increase of relative diameter. The wooden and steel intruders on the other hand were rising faster for larger intruders. The wooden intruder is still situated on the density peak. Its rise time is therefore only slightly decreasing, whereas the rise time of the steel intruder is dropping very quickly for larger intruders. In the same graph it can be observed that for smaller intruders ($D/d \ll 10$) the rise time is getting equal to the rise time of the background material.

-Multi intruder[2]:
Exactly the same experiments have been performed with multiple intruders, but now the diameter of the individual intruders remained constant and only the number of intruders is increased. We observed that most of the time the intruders stayed in the configuration they were put in at a depth $z_0=4.5cm$, thus acting as a compound. So we can speak of increasing the size of the compound when the number of intruders is increased! This means that no interactions between the intruders are taking place.

For all experiments in both 0.5mm and 1mm glass beads we used the same default configurations as shown in figure 10. In all of these configurations the intruders were initially touching each other to reassure a smallest area.

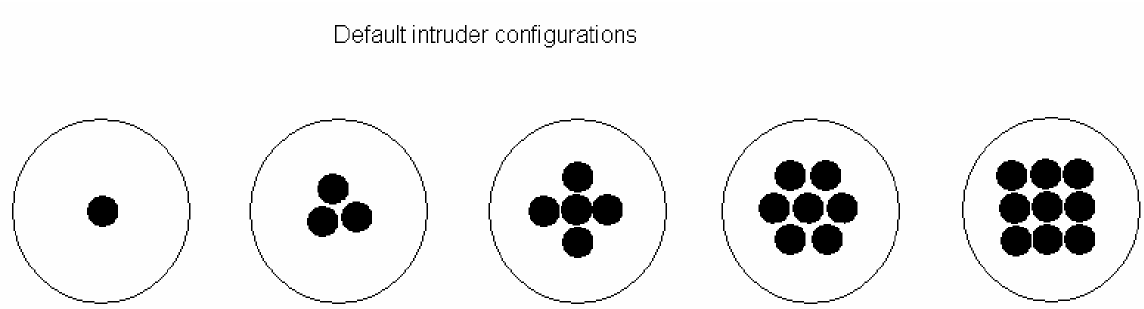


Fig 10: The default configurations used for all the multi intruder experiments with 1,3,5,7 and 9 intruders. These configurations were put at a depth z_0=4.5 cm below surface in a horizontal plane. After putting in the configuration the size dependence experiment was carried out.

[2] The rise time, T_{rise}, is defined as the number of taps applied to the system for the first intruder to appear at the surface. Normally the other intruders of the configuration emerged in 3 through 5 taps after the first one and are therefore considered to act as a compound.

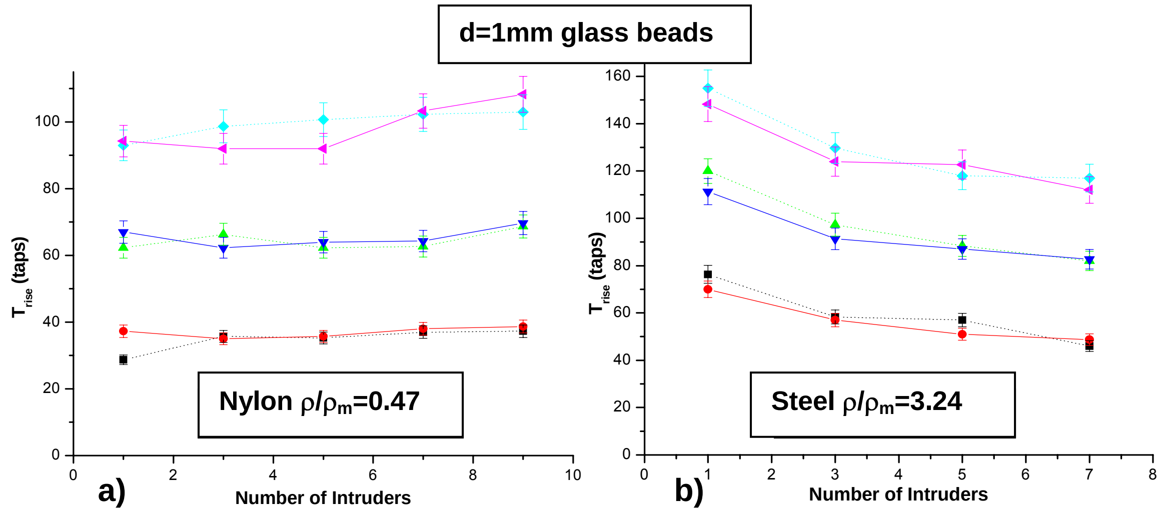


Fig 11: Multi size dependence results for 1mm glass beads using nylon (fig.11a) and steel intruders (fig.11b) of size D/d=19. For both graphs applies:

- *Depth z_0=7cm: atmospheric pressure (solid ◄), lower pressure (dotted ♦)*
- *Depth z_0=4.5cm: atmospheric pressure (solid ▼), lower pressure (dotted ▲)*
- *Depth z_0=2cm: atmospheric pressure (solid ●), lower pressure (dotted ■)*

Due to surfacing problems with the steel intruders is the lower pressure (dotted) different for the nylon and the steel occasion: for nylon (fig.11a) is the lower pressure P=0.053 kPa and for steel (fig.11b) P=22.7 kPa.

The results for 1mm glass beads are shown in figure 11. No clear trend can be obtained from the graph in figure 11a for nylon, because the rise time remains approximately the same. This applies for the experiments at atmospheric pressure and at lower pressure, in which there is barely a difference between these two series of experiments.

A comment has to be made about the fluctuations in the taken measurements, because the nylon intruder data was fluctuating much more than the data obtained with steel ones. Just as with the single intruder experiments this effect is due to the fact that nylon is near the density dependence peak.

For the steel intruders (figure 11b) it is obvious that the rise time is decreasing when the number of intruders (the size of the compound) is increased. This suits the picture of the single size dependence graph (figure 9) for steel perfectly, where the largest intruders also rise fastest. Like for nylon, there is no large difference between the experiment performed under a lower pressure and the one at atmospheric pressure.

For 1mm background material in general it can be observed from figures 11a and 11b that there is no depth dependence for the size dependence. After all the size dependence graph shows approximately the same trend at every depth. These graphs also show the decreasing velocity of the intruder(s) while rising to the surface and especially the upper 2cm takes longest. The reason could be that the top layer is more fluidized, so the steel intruder sinks back in this layer after each tap. This causes a significant decrease of the velocity of the intruders in the top layer of approximately 1~2cm.

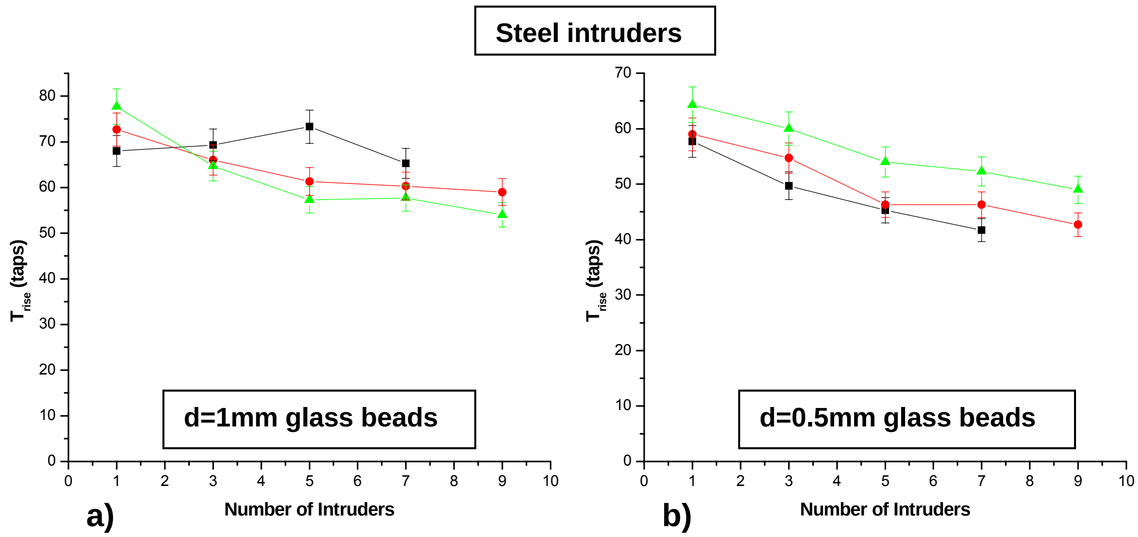


Fig 12: Multi size dependence result using different sizes of steel (ρ/ρ_m=3.24 and ρ/ρ_m=3.10 respectively) intruders in 1mm background material (fig.12a) and 0.5mm background (fig.12b) at atmospheric pressure.

■ – *1" intruder(s)*
● – *¾" intruder(s)*
▲ – *½" intruder(s)*

In figure 12[3] the result of the experiment with three different sizes of steel intruders can be seen. It seems that in 1mm background (figure 12a) the smaller intruders are affected more than the larger ones if the number of intruders is increased. For the ½" and ¾" intruders the rise time is clearly decreasing, but the rise time for 1" steel intruder seems to be constant. The experiment with nine 1" steel intruders has not been performed, because the wall of the cylinder affects the intruders.

A transition can even be observed in figure 12a around two intruders. Beyond this transition the largest intruders are suddenly slowest and this is in contradiction with the single intruder relation. With single intruders the largest intruders are rising fastest, but more experiments are necessary to verify the existence of this transition.

For 0.5mm glass beads (figure 12b) the rise time is decreasing for all sizes of intruders if the number of intruders is increased. Here the largest intruders stay rising fastest no matter how many intruders are put in the cylinder.

[3] The apparent inconsistency in the range of rise times in two equal experiments, which are depicted in figures 11b (solid ▼) and 12a (●), is due to a changed definition of the rise time. Because of the problems of the steel intruders surfacing in 1mm glass beads mentioned earlier, we adopted the next definition of rise time *only* for this occasion: T_{rise} is the time the first glance of the intruder(s) (the dimple in surface) can be seen.

The different configurations of ¾” steel intruders at a depth of $z_0=4.5cm$ are rising a factor two faster than in the 1mm glass beads (see figure 11b, solid ▼). Due to the smaller glass beads is the relative diameter ratio (*D/d*) also a factor two larger than for 1mm glass beads.

The graph shown in figure 12b directed us to the idea of an effective diameter of the compound, i.e. a relation between the three sizes of steel intruders used. So we analyzed this data of figure 12b more profoundly and produced the two graphs in figure 13.

d=0.5mm glass beads

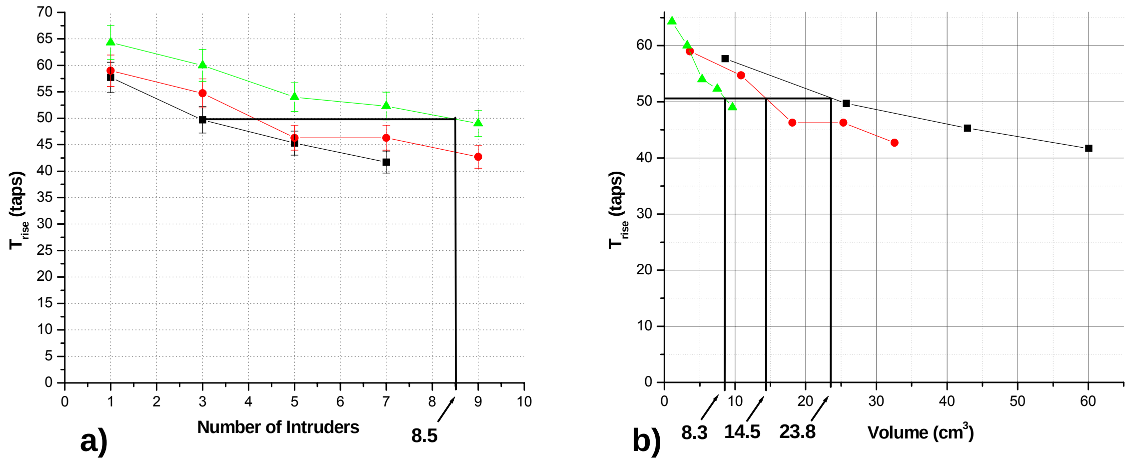


Fig 13: To determine the effective diameter these two representations of the data in 0.5mm glass beads were useful.

■ – *1” intruder(s)*
● – *¾” intruder(s)*
▲ – *½” intruder(s)*

In order to obtain this relation for the effective diameter many linking lines were drawn in these graphs. For each graph in figure 13 an example line is drawn and the one in figure 13a means; three 1” steel intruders rise as fast as 8.5 ½” steel intruders. So a factor of 2.8 is found for this line linking the 1” to the ½” intruders. The same was done for the other possible links between all sizes of intruders yielding the next ‘rule of thumb’ for the effective diameter:

1 1” intruder ~ 1.5 ¾” intruders ~ 3.1 ½” intruders *(2)*

In other words;

- For every **1”** steel intruder, the number of **¾”** intruders needed to get the same rise time is approximately: **1.5**
- For every **1”** steel intruder, the number of **½”** intruders needed to get the same rise time is approximately: **3.1**
- For every **¾”** steel intruder, the number of **½”** intruders needed to get the same rise time is approximately: **2.0**

These figures really have to be regarded as a rule of thumb, because we are assuming a linear behavior. Much more experimental data is necessary for a good and more significant comparison.

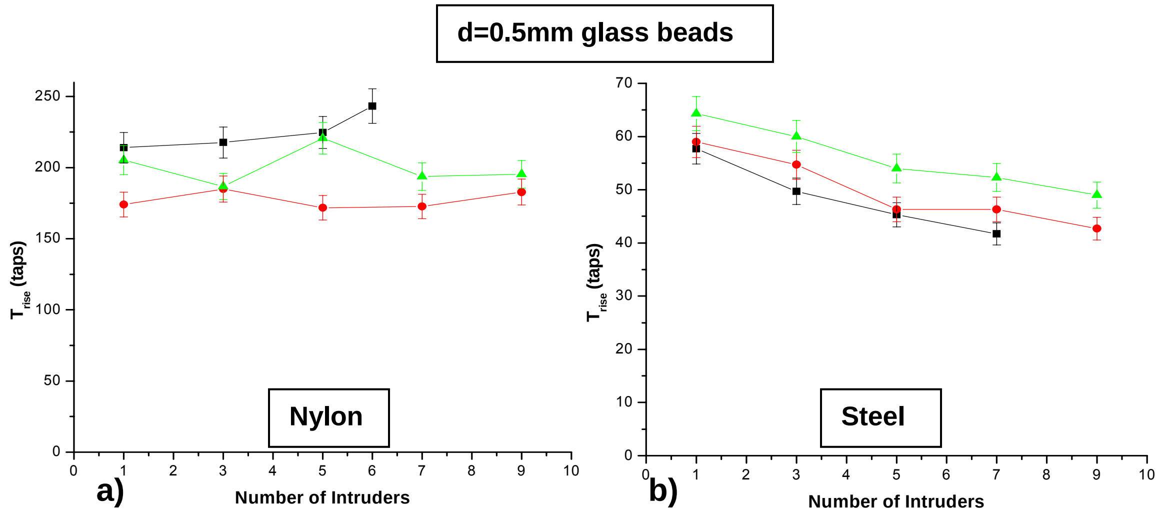


Fig 14: Multi size dependence in 0.5mm glass beads using nylon and steel intruders (ρ/ρ_m=0.45 and ρ/ρ_m=3.10 respectively). For both graphs applies:

- ■ – *1" intruder(s) (D/d=51)*
- ● – *¾" intruder(s) (D/d=38)*
- ▲ – *½" intruder(s) (D/d=25)*

One has to be careful with the nylon result depicted in figure 14a, because the configurations containing more than 5 intruders break up and cannot be considered as a 'compound' anymore.

Regarding the results of figure 14a it has to be mentioned firstly that they have to be taken with great cautiousness. We encountered large and, more important, unsolvable problems regarding the rising nylon intruders. Up to 5 nylon intruders in a configuration were rising and emerging perfectly in the same configuration they were put in. Any configuration over 5 intruders was not rising at the same velocity anymore. Every time two or more intruders stayed behind, while the rest of the configuration rose as they were inserted. So only part of the configuration was neatly emerging at the surface, whereas the others slowed down enormously or even sank to the bottom! A lot of approaches have been tried, but none made the larger configurations emerge as a 'compound'. So we have to be careful with this graph.

At least we can conclude from figure 14a that the largest intruders are slowest for the nylon intruders, which is opposite to the same experiment with steel intruders (figure 14b). Just as in the 1mm background case, the rise time remains approximately the same for all sizes of nylon intruders. The rise time is a factor two to three slower than the configurations in the 1mm background material too. Again this effect can be attributed to the fact that due to smaller glass beads the relative diameter ratio (*D/d*) is a factor two larger.

Miscellaneous:

-Depth dependence:
To determine the depth dependence in the cylinder, the intruder was placed at different depths, in this way obtaining the depth dependence as shown in figure 15.

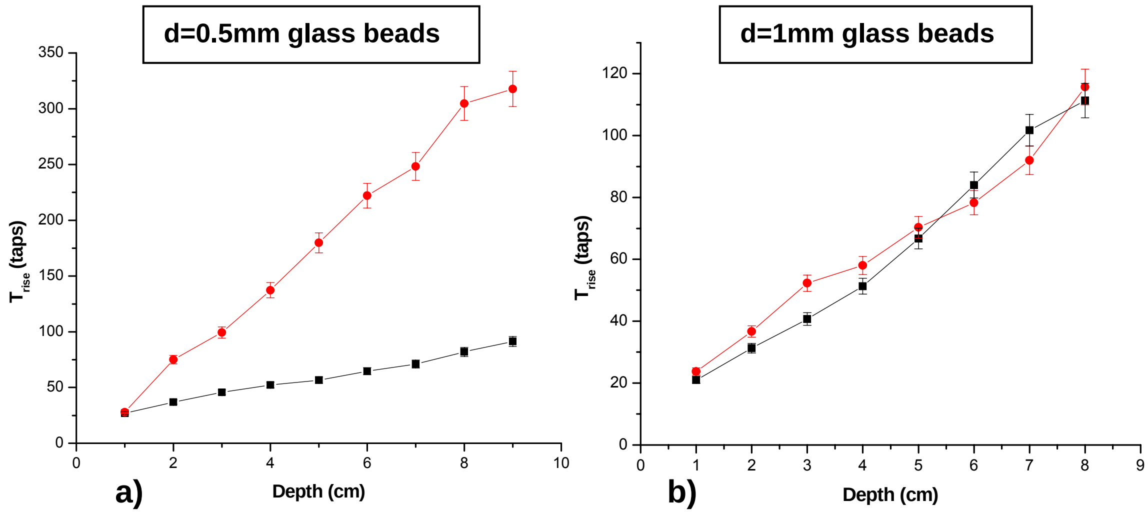


Fig 15: Depth dependence determined for both background materials using a 1” steel and nylon intruder. For both graphs applies;

■ – *1” steel intruder ($\rho/\rho_m = \pm 3.1$)*

● – *1” nylon intruder ($\rho/\rho_m = \pm 0.5$)*

In the 0.5mm case (figure 15a) the relation is almost linear for both the nylon and the steel intruder, but steel is much faster than nylon. This can be attributed to the fact that the nylon intruder is on the peak in rise time of the density dependence graph. Therefore the nylon intruder is much slower than the steel one in 0.5mm glass beads.

For the 1mm glass beads in figure 15b however, the nylon intruder is as fast as the steel one. Because the density dependence ‘peak’ for 1mm glass beads is only a little wiggle and not as pronounced as in the 0.5mm glass beads case. This caused much larger fluctuations in measurements for nylon than for the steel intruder too.

It can be observed in figure 15b that both intruders are rising at a constant rate up to the top layer of 1~2cm. The top 1 or 2 cm is more loosely packed and caused the intruder to sink back more in the glass beads than in the lower layers. The intruders were still effectively moving up causing a relatively longer rise time in this upper layer for the intruders in both background materials. This is in agreement with the results depicted in figure 11.

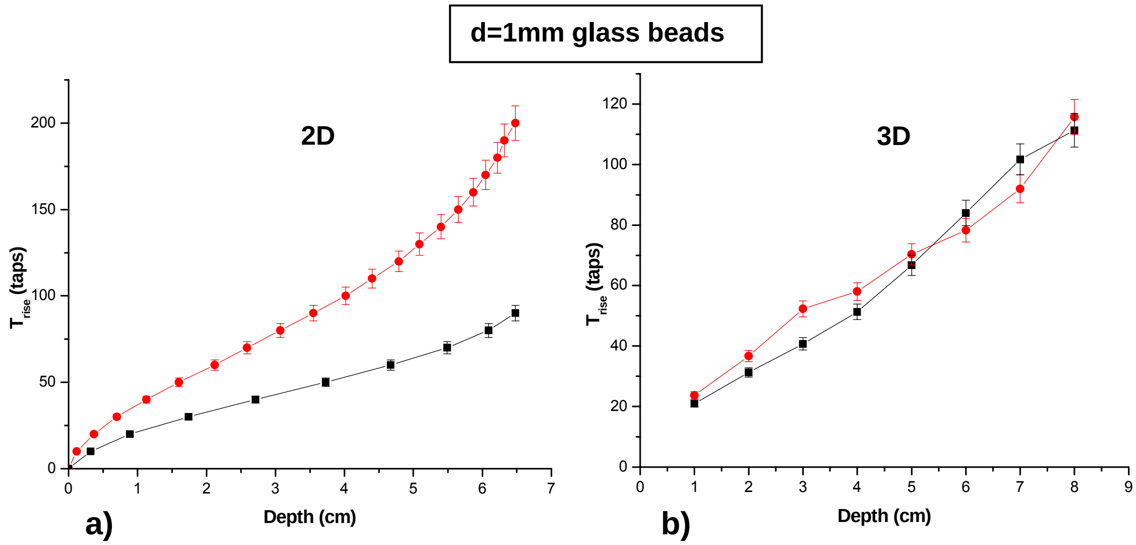


Fig 16: Depth dependence in the semi-2D situation [5] and 3D situation. In both situations 1mm glass beads were used at atmospheric pressure.

■ – *1” steel disk (ρ/ρ_m= 3.05)*
● – *1” steel ring (ρ/ρ_m= 0.37)*

■ – *1” steel intruder (ρ/ρ_m= 3.24)*
● – *1” nylon intruder (ρ/ρ_m= 0.47)*

The comparison of figure 16 between the semi-2D situation of Niemuth et al. [5] and the 3D situation shows that the disk and ring do not stay together. Where in the 3D situation the nylon and steel intruder do stay together. The semi-2D container is particularly convenient to extract a depth dependence graph with good resolution as can be seen in figure 16a. From this figure it can be concluded that the disk and ring rise fastest in the middle of the container. The 3D situation of figure 16b cannot provide the same information due to practical boundaries such as the larger fluctuations in the measurements.

-Filling height dependence:
In order to check if a different filling height affected the results of the experiments performed a lower filling height of *h=7cm* was used to produce figure 17.

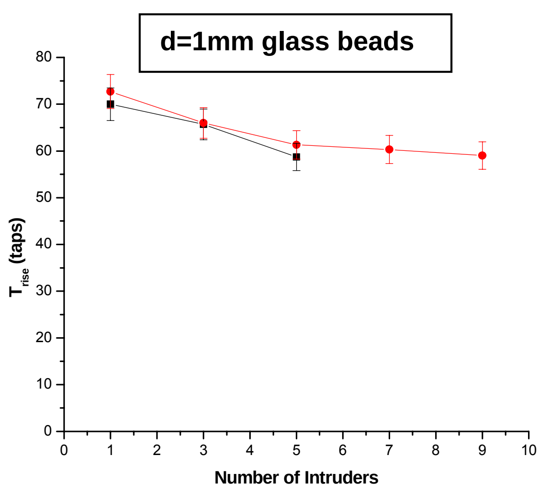


Fig 17: Filling height dependence using ¾” steel intruders (ρ/ρ_m= 3.24) in 1mm glass beads at atmospheric pressure. All configurations were still put at a depth z_0=4.5 cm below surface.

■ – *Lower filling height: h=7cm*
● – *Normal filling height: h=11cm*

Although the result shown in figure 17 is just a quick check, it surely shows that for 1mm glass beads the result is not affected by a different filling height. When reasonable filling heights are considered, for which the bottom effects are excluded, no affection of the results are assumed for both background materials.

-Configuration dependence:
The default configurations shown in figure 10 always have the smallest possible area. To investigate the dependence on different configurations, the different configurations with 3 intruders from figure 18 were used.

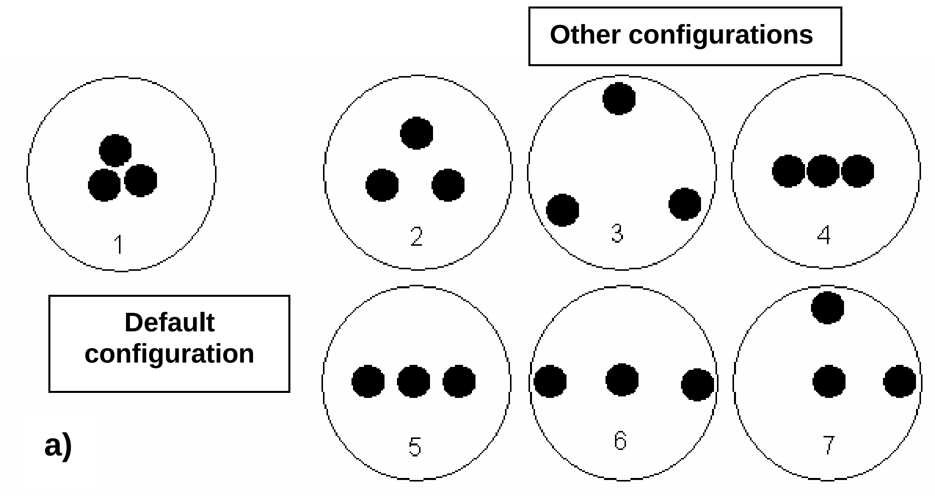
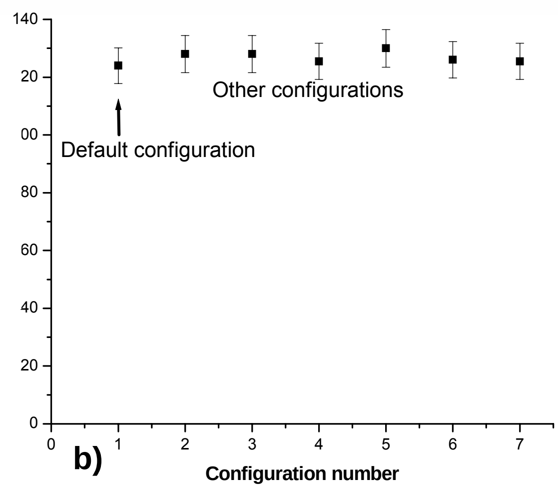


Fig 18: The 7 different configurations (fig.18a) used in order to determine the configuration dependence (fig.18b). Three ¾” steel intruders (ρ/ρ_m= 3.24) in 1mm glass beads at depth z_0=7cm were used.



The resulting graph of figure 18b shows no significant difference for the rise time between the six different configurations from figure 18a and the default configuration. So for configurations containing 3 intruders no dependence on different configurations has been found.

-Interesting phenomenon – 3 intruders vertical:
During the configuration dependence experiment we tried out something different. Instead of putting 3 intruders in a horizontal plane we put them vertically

in the center. Many different vertical configurations have been investigated and a very interesting phenomenon occurred, which is shown in figure 19.

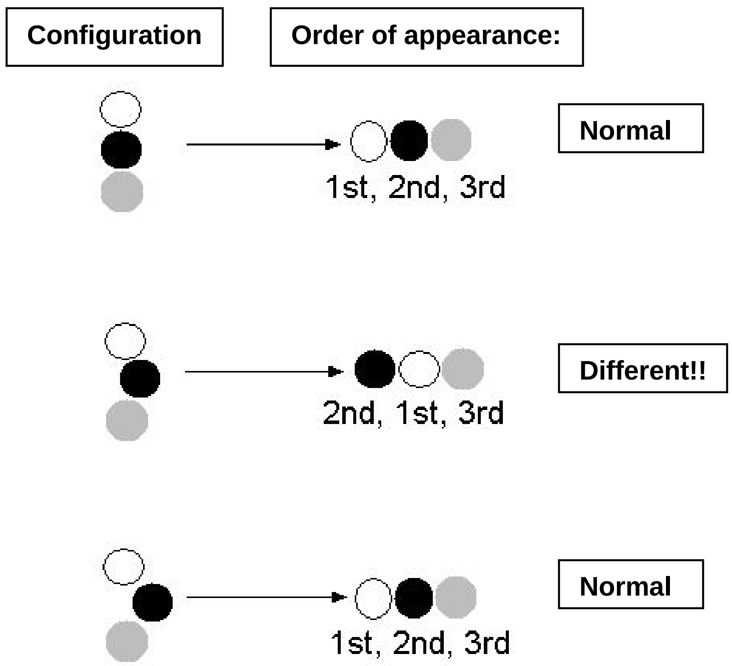


Fig 19: An interesting phenomenon occurred with three ¾” steel intruders (ρ/ρ_m= 3.24) in a vertical configuration. The first intruder was put roughly at depth z_0=7cm in 1mm glass beads. The system is very sensitive to the initial offset of the 2nd intruder. An offset of ≈½radius yielded the effect of the 2nd intruder catching up with the 1st one

For the experiments without any offset of the 2nd intruder and with a ‘large’ offset the order of appearance was the normal and expected order. So the one on top appeared first, followed by the middle intruder and the one near the bottom. Unexpectedly with a small offset of approximately *½radius* the 2nd intruder was able to catch up with the first one. In most of the experiments performed with this offset the 2nd intruder emerged first, quickly followed by the 1st intruder. But we have to be very careful with this result, because the system is very sensitive to the initial offset. Sometimes the first intruder appeared just before number two or they emerged at the same time.

Conclusions & Discussion

Density dependence:

The effect of a different density intruder is not as significant for 1mm glass beads as for the 0.5mm background material. In the 0.5mm case the rise time has peak around $\rho/\rho_m \approx 0.5$, which is a factor 3 higher than $T_{asymptote}$. The reason for this sudden increase *and* drop of the rise time is still unknown. For 1mm glass beads only a small unstable region can be observed at $\rho/\rho_m \approx 0.5$, but T_{rise} can be considered as slightly increasing.

Size dependence:

The single as well as the multi intruder experiments (for both glass beads) show for intruders far from the density peak; a larger single intruder or a larger configuration acting as a compound rises faster.

For both the single and multi size experiment applies that intruders situated on this peak (nylon) show the same rise time for all sizes if 1mm glass beads are used. In the 0.5mm case the single intruder rises slower if the diameter is increased. For these glass beads the multi nylon experiment is highly unstable (not considered as a compound anymore) and no clear trend can be obtained from the data. These effects can be attributed to an unstable region around the density peak.

So increasing the number of intruders in the configuration can be considered as enlarging the effective diameter of the compound. These effective diameters can be linked to get the same rise time using different sizes of intruders. The next 'rule of thumb' relates 3 different sizes of steel intruders to obtain the same T_{rise}:

1 1" intruder ~ 1.5 ¾" intruders ~ 3.1 ½" intruders

Miscellaneous:

-Depth dependence: Considered to be linear up to the top layer of 1~2cm, where the wooden and steel intruders are slowing down. This effect is due to the fact that the top layer is more loosely packed causing the intruders to sink back more every tap, but still moving up effectively.

-Filling height dependence: A different filling height does not seem to affect the results of the experiments performed. More data from all experimental situations is required to check this dependence more profoundly.

-Configuration dependence: Using different configurations containing three intruders did not affect the rise time significantly in our experiment. The same

experiment needs to be performed with more than three intruders to be sure for all number of intruders.

-Interesting phenomenon – 3 intruders vertical: When three intruders were placed vertically and the 2nd intruder is given an offset of *≈½radius*, this 2nd intruder is able to catch up with the 1st one. The system is extremely sensitive to the initial offset, so this result has to be treated with great cautiousness.

Semi-2D vs. 3D:

The semi-2D density result is not in agreement with the 3D situation; the rise time is decreasing for denser intruders in the 2D container. Where it is slightly increasing for the comparable 1mm glass beads 3D situation. For 0.5mm glass the difference is even more obvious: a non-monotonic density dependence was found in the 3D case versus a monotonic one in 2D.

Regarding all the results of [5] only the depth dependence of the 2D situation agrees with the 3D situation. In the 3D situation the resolution is lower due to practical problems, but it seems to be the same as in the 2D situation; the intruder is fastest in the middle of the container and definitely slowing down in the top layer.

General:

From all our experiments for both glass beads can be concluded that the role of interstitial air is very important. Every time the occurring effects were far more pronounced with the 0.5mm glass beads than in the 1mm background medium. This was clearly visible in the density and size dependence experiments.

Recommendations

All the important parameters involved in the Brazil Nut Problem have been investigated right now, but the exact interactions causing this behavior are still to be revealed. This can be done by 3D-flow visualization experiments using magnetic resonance imaging (MRI), which can show the interactions happening inside of the 3D cylinder. With MRI the vertical velocity flow profile of a cylinder without an intruder has already been determined by J.B. Knight et al. [10]. This method can perfectly be used to find the flow in a cylinder with an intruder present. In this way the reasons for the effects mentioned in this report can be tried to figure out.

As always in Granular Matter more data in general is needed to get more significant results regarding all experiments performed. Particularly the experiments performed only at atmospheric pressure have to be investigated in the lower pressure case. Further experiments have to be done on the effective diameter of the different configurations to improve the 'rule of thumb' too. Additional experiments have to be carried out on the filling height and configuration dependence especially in 0.5mm glass beads. Plus of course the interesting phenomenon with three intruders vertically has to be checked in this medium as well.

Besides providing more new data for experiments already performed, some new experiments can be performed as well:

- By varying the viscosity, ν, in the 0.5mm glass beads case it can be tried to get back to the regime of 1mm glass beads. These less pronounced effects of the 1mm background material can be tried to accomplish by filling the cylinder with helium after evacuating all the air.
- In all experiments the same intruders (same size and density) have been used up till now. It can be very interesting to see what happens when different size and/or density intruders are used.

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